Excitation of Surface Waves on a Perfectly Conducting Screen Covered with Anisotropic Plasma*

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Summary—**The field due to a line source of magnetic current situated in a lossless plasma region above a perfectly conducting screen is considered when a uniform static magnetic field is impressed throughout the plasma region parallel to the direction of the line source. It is shown that under certain conditions surface waves are excited on the screen. The dependence of the efficiency of excitation of surface waves on the distance *d* of the line source from the ground screen is examined. Also, the asymptotic series for the radiation field is derived, and its leading term is shown to vanish for a particular value of *d*. Under these conditions a strong surface-wave field is maintained near the guiding surface.**

Introduction

In a recent paper,[1] surface waves were shown to exist on a screen which is assumed to be perfectly conducting in a given direction and completely insulating in the perpendicular direction. In this paper, similar surface waves are shown to exist even when the screen is perfectly conducting provided the medium is anisotropic. Specifically, when a line source of magnetic current is situated in a lossless plasma above a perfectly conducting screen and a uniform static magnetic field is impressed throughout the plasma parallel to the line source, surface waves can exist on the screen. The surface waves are generated only when the plasma is anisotropic and when the operating frequency exceeds the plasma frequency. In addition, for a particular ratio of the operating to the plasma frequency, the static magnetic field must be less than a critical value. The efficiency of excitation of surface waves is evaluated and its dependence on the distance d of the line source from the ground screen is examined for one set of values of plasma and gyromagnetic frequencies. Further, it is shown that by a proper choice of d, it is possible to nullify the leading term in the asymptotic series for the radiation field and thereby obtain a surface wave field which is much stronger than the radiation field near the guiding surface. Similar results have been obtained for the case of surface waves excited by a line source of magnetic current on a dielectric-coated conducting screen.[2]

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[1] S. R. Seshadri, "Excitation of surface waves on a unidirectionally conducting screen," IRE Trans. on Microwave Theory and Techniques, vol. 10, pp. 279–286; July, 1962.

[2] A. L. Cullen, "The excitation of plane surface waves," *Proc. IEE*, Monograph No. 93R, vol. 101, p. 225; February, 1954.

Excitation of Surface Waves

Consider a perfectly conducting screen of infinite extent located in the x-y plane, where x, y, and z form a right-handed coordinate system. The half space $z>0$ is filled with a uniform plasma. In this investigation, only a primitive model is assumed for the plasma; that is, 1) the plasma as a whole is considered to be at rest, 2) the pressure gradient in the equation of motion is neglected in comparison with the effect of the alternating electric field, 3) the plasma is assumed to be lossless, and 4) the oscillations of the ions are neglected in comparison with those of the electrons.

A line source of magnetic current is located in the plasma at $x=0$, $z=d$; it is parallel to the y axis and may be represented as

$$J_m = \hat{y}\delta(x)\delta(z-d). \tag{1}$$

A uniform magnetic field B_0 is impressed in the y direction throughout the plasma. See Fig. 1. It is desired to examine the electromagnetic field set up by the line source inside the plasma.

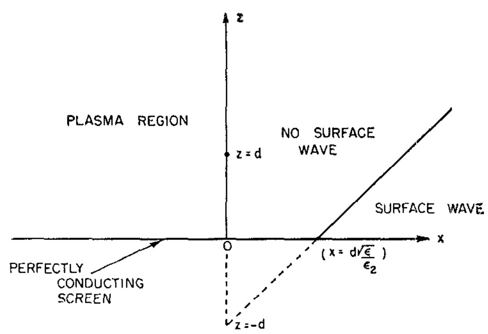


Fig. 1—Geometry of the problem.

As a consequence of the primitive model assumed for the plasma, it is found that in the plasma region ($z>0$), after the usual linearization, the electric and magnetic fields satisfy the time-harmonic Maxwell's equations

$$\nabla \times \boldsymbol{E} = i\omega\mu_0 \boldsymbol{H} - J_m \tag{2a}$$

$$\nabla \times \boldsymbol{H} = -i\omega\epsilon_0 \boldsymbol{\varepsilon}\cdot\boldsymbol{E} \tag{2b}$$

where μ_0 and ϵ_0 are the permeability and dielectric constant pertaining to vacuum. A harmonic time dependence $e^{-i\omega t}$ is assumed for all the field components. The components of the relative dyadic dielectric constant

$\boldsymbol{\varepsilon}$ are given by the following matrix

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \epsilon_1 & 0 & -i\epsilon_2 \\ 0 & \epsilon_3 & 0 \\ i\epsilon_2 & 0 & \epsilon_1 \end{bmatrix} \tag{3}$$

where

$$\epsilon_1 = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \left[1 - \left(\frac{\omega_c}{\omega}\right)^2\right]^{-1}$$

$$\epsilon_2 = \left(\frac{\omega_p}{\omega}\right)^2 \left[\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega}\right]^{-1}$$

and

$$\epsilon_3 = 1 - \left(\frac{\omega_p}{\omega}\right)^2. \tag{4}$$

The plasma frequency ω_p and the gyromagnetic frequency ω_c of the electrons are given by

$$\omega_c = -\frac{eB_0}{m}, \qquad \omega_p{}^2 = \frac{ne^2}{m\epsilon_0} \tag{5}$$

where e is the charge of an electron, m is the mass of an electron, n is the electron density, and B_0 is the applied static magnetic field.

The source and the geometry of the problem are independent of the y coordinate and therefore, all the field quantities are invariant with respect to the y coordinate. With $\partial/\partial y \equiv 0$ in (2), the electromagnetic field is separable into E and H modes which are excited, respectively, by line sources of magnetic and electric current. Since only a line source of magnetic current is present, the H mode is not present; and hence, $E_y = H_x = H_z = 0$. Only a single component of the magnetic field, namely, H_y, is present. It is easily shown with the help of (2b) that the remaining components of the electric field are given by

$$E_x(x, z) = -\frac{i\epsilon_1}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial z}H_y(x, z) - \frac{\epsilon_2}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial x}H_y(x, z)$$

$$E_z(x, z) = \frac{i\epsilon_1}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial x}H_y(x, z) - \frac{\epsilon_2}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial z}H_y(x, z) \tag{6}$$

where

$$\epsilon = \epsilon_1{}^2 - \epsilon_2{}^2. \tag{7}$$

With the help of (2a) and (6) it follows that $H_y(x, z)$ satisfies the following source-dependent wave equation:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] H_y(x, z) = -\frac{i\omega\epsilon_0\epsilon}{\epsilon_1}\delta(x)\delta(z - d) \tag{8}$$

where

$$k^2 = \frac{\omega^2\mu_0\epsilon_0\epsilon}{\epsilon_1} = \frac{k_0{}^2\epsilon}{\epsilon_1}. \tag{9}$$

In (9), k_0 is the wave number corresponding to vacuum. It is assumed in what follows that $\epsilon > 0$. This requirement imposes certain restrictions on the range of values of ω_p/ω and ω_c/ω.

The geometry of the problem suggests the following representations for the field components:

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{H}_y(\zeta, z)e^{i\zeta x}d\zeta \tag{10a}$$

$$E_x(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{E}_x(\zeta, z)e^{i\zeta x}d\zeta \tag{10b}$$

and

$$E_z(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{E}_z(\zeta, z)e^{i\zeta x}d\zeta. \tag{10c}$$

It follows from (6), (8) and (10) that

$$\overline{E}_x(\zeta, z) = -\frac{i\epsilon_1}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial z}\overline{H}_y(\zeta, z) - \frac{i\epsilon_2\zeta}{\omega\epsilon_0\epsilon}\overline{H}_y(\zeta, z), \tag{11a}$$

$$\overline{E}_z(\zeta, z) = -\frac{\epsilon_1\zeta}{\omega\epsilon_0\epsilon}\overline{H}_y(\zeta, z) - \frac{\epsilon_2}{\omega\epsilon_0\epsilon}\frac{\partial}{\partial z}\overline{H}_y(\zeta, z), \tag{11b}$$

and

$$\left[\frac{d^2}{dz^2} + \xi^2\right]\overline{H}_y(\zeta, z) = -\frac{i\omega\epsilon_0\epsilon}{\epsilon_1}\delta(z - d) \tag{12}$$

where

$$\begin{aligned} \xi &= +\sqrt{k^2 - \zeta^2} \qquad & k > \zeta \\ \xi &= +i\sqrt{\zeta^2 - k^2} \qquad & k < \zeta. \end{aligned} \tag{13}$$

The solution of (12) gives

$$\overline{H}_y(\zeta, z) = \begin{cases} Ae^{i\xi z} + Be^{-i\xi z} & d < z \\ Ce^{i\xi z} + De^{-i\xi z} & d > z \end{cases} \tag{14}$$

and

$$\frac{d}{dz}\overline{H}_y(\zeta, d+) - \frac{d}{dz}\overline{H}^v(\zeta, d-) = -\frac{i\omega\epsilon_0\epsilon}{\epsilon_1}. \tag{15}$$

The radiation condition requires $H_y(x, z)$ to be an outgoing wave for $z \to \infty$; hence $B = 0$. Since the tangential component of the electric field is zero for $z = 0$, it follows from (11a) and (14) that

$$C = R(\zeta)D \quad \text{and} \quad R(\zeta) = \frac{\epsilon_1\xi + i\epsilon_2\zeta}{\epsilon_1\xi - i\epsilon_2\zeta}. \tag{16}$$

The requirement that the tangential component of the magnetic field should be continuous at $z = d$ gives

$$Ae^{i\xi d} = Ce^{i\xi d} + De^{-i\xi d}. \tag{17}$$

The use of the jump condition (15) in (14) leads to

$$Ae^{i\xi d} - (Ce^{i\xi d} - De^{-i\xi d}) = -\frac{\omega\epsilon_0\epsilon}{\epsilon_1\xi}. \tag{18}$$

The expressions for A, C and D may be obtained from the solution of the simultaneous equations (16), (17) and (18). The results are:

$$A = -\frac{\omega\epsilon_0\epsilon}{2\epsilon_1\xi}\left[e^{-i\xi d} + R(\zeta)e^{i\xi d}\right],$$

$$B = 0,$$

$$C = -\frac{\omega\epsilon_0\epsilon}{2\epsilon_1\xi}R(\zeta)e^{i\xi d},$$

$$D = -\frac{\omega\epsilon_0\epsilon}{2\epsilon_1\xi}e^{i\xi d}. \tag{19}$$

The substitution of (14) (10a) and the use of (16) and (19) yields

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{\omega\epsilon_0\epsilon}{2\epsilon_1\xi}\left[e^{-i\xi d} + \frac{\epsilon_1\xi + i\epsilon_2\zeta}{\epsilon_1\xi - i\epsilon_2\zeta}e^{i\xi d}\right] \cdot e^{i\zeta x + i\xi z}d\zeta \qquad \text{for } z > d. \tag{20a}$$

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{\omega\epsilon_0\epsilon}{2\epsilon_1\xi}\left[e^{-i\xi z} + \frac{\epsilon_1\xi + i\epsilon_2\zeta}{\epsilon_1\xi - i\epsilon_2\zeta}e^{i\xi z}\right] \cdot e^{i\zeta x + i\xi d}d\zeta \qquad \text{for } z < d. \tag{20b}$$

The contour for the integrals in (20a) and (20b) is along the real axis in the ζ plane as shown in Fig. 2. The integrand in (20a) and (20b) has a pole at

$$\zeta = +\frac{k\epsilon_1}{\sqrt{\epsilon}} = +k_0\sqrt{\epsilon_1}.$$

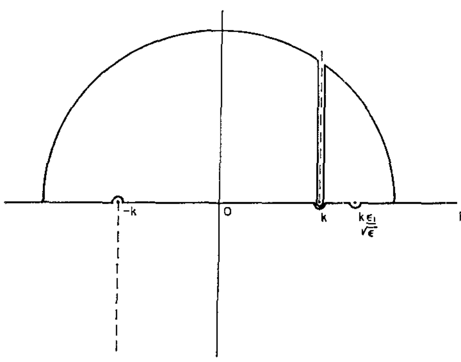


Fig. 2—Contour of integration in the ζ plane

This pole is either on the real or the imaginary axis, depending upon whether ϵ_1 is positive or negative. Since the aim of this paper is to examine the characteristics of the surface wave that is excited on the ground screen, only positive values of ϵ_1 are considered. This leads to the following restrictions:

1) $\frac{\omega_p}{\omega} < 1$ and

2) $-\left[1 - \left(\frac{\omega_p}{\omega}\right)^2\right]^{1/2} < \frac{\omega_c}{\omega} < \left[1 - \left(\frac{\omega_p}{\omega}\right)^2\right]^{1/2}.$

For $0 < \epsilon_1 < 1$, the pole of the integrand is on the real axis and the contour of integration is indented above the singularity $-k$ and below the singularities at k and $k\epsilon_1/\sqrt{\epsilon}$. For $x > 0$, the integrals may be evaluated by closing the contour in the upper half of the ζ plane as shown in Fig. 2. The contribution to the integrals in (20a) and (20b) is the sum of the residue at the pole $\zeta = k\epsilon_1/\sqrt{\epsilon}$ and a branch-cut integral.

The value of the branch-cut integral depends on some inverse power of x. Hence, for sufficiently large x, the value is negligible compared to the contribution due to the pole. Thus, for positive large x,

$$H_y(x, z) = -\omega\epsilon_0|\epsilon_2| \exp\left\{\frac{ik\epsilon_1}{\sqrt{\epsilon}}x - \frac{k|\epsilon_2|}{\sqrt{\epsilon}}(z + d)\right\}$$
$$= -\omega\epsilon_0|\epsilon_2| \exp\left\{ik_0\sqrt{\epsilon_1}\,x - \frac{k_0|\epsilon_2|}{\sqrt{\epsilon_1}}(z + d)\right\}. \tag{21}$$

It is obvious that $H_y(x, z)$ given in (21) represents a surface wave propagating in the positive x direction and exponentially attenuated in the z direction. The phase velocity v_p of the wave is given by

$$v_p = \frac{c_0}{\sqrt{\epsilon_1}} = \frac{c\sqrt{\epsilon}}{\sqrt{\epsilon_1}} \tag{22}$$

where c_0 and c are, respectively, the velocity of propagation of electro-magnetic waves in vacuum and in an unbounded anisotropic plasma. Since $\epsilon_1 < 1$, the phase velocity of the surface wave is greater than the velocity of electromagnetic waves in vacuum and less than that in the unbounded anisotropic plasma. When the static magnetic field is reversed in direction, ϵ_2 changes sign and the pole of the integrand in (20a) and (20b) occurs now at $\zeta = -k\epsilon_1/\sqrt{\epsilon} = -k_0\sqrt{\epsilon_1}$. Hence, the surface wave reverses direction and propagates in the negative x direction, instead of in the positive x direction as described in (21). When there is no static magnetic field, ϵ_2 vanishes and from (20a) and (20b) it is clear that, in this case, there is no surface wave. This means that besides other restrictions the plasma must be anisotropic if surface waves are to exist on the ground screen.

The substitution of (21) in (6) gives the other field components of the surface wave as follows:

$$E_x(x, z) = 0$$

$$E_z(x, z) = \frac{k|\epsilon_2|}{\sqrt{\epsilon}} \exp\left\{\frac{ik\epsilon_1}{\sqrt{\epsilon}}x - \frac{k|\epsilon_2|}{\sqrt{\epsilon}}(z + d)\right\}. \tag{23}$$

Therefore, the surface wave is a TEM wave with respect to the direction of propagation.

In the integral representation (20a) and (20b) for $H_y(x, z)$, the first term corresponds to the incident field due to the line source and the second term corresponds to the field of the line source reflected from the ground screen. The asymptotic form of the reflected field is obtained by performing a saddle-point evaluation of the integral (20a) and (20b). For this purpose, the transformation

$$\zeta = k \cos \tau \tag{24}$$

is introduced. With it, the expression [(20a) and (20b)] for $H_y(x, z)$ becomes

$$H_y(x, z) = H_y{}^i(x, z) + H_y{}^r(x, z)$$

where

$$H_y{}^i(x, z) = \frac{1}{2\pi}\int_C \frac{\omega\epsilon_0\epsilon}{2\epsilon_1} e^{ik[x \cos \tau + |z-d| \sin \tau]} d\tau \tag{26}$$

$$H_y{}^r(x, z) = \frac{1}{2\pi}\int_C \frac{\omega\epsilon_0\epsilon}{2\epsilon_1} \frac{\epsilon_1 \sin \tau + i\epsilon_2 \cos \tau}{\epsilon_1 \sin \tau - i\epsilon_2 \cos \tau} \cdot e^{ik[x \cos \tau + (z+d) \sin \tau]} d\tau. \tag{27}$$

The original contour along the real axis in the ζ plane is transformed into the contour C shown in Fig. 3. The asymptotic form of the reflected field is obtained by a saddle-point evaluation of the integral in (27). The saddle point which lies in the interval $0<\tau_0<\pi$ is given by

$$\tau_0 = \tan^{-1} \frac{z + d}{|x|} \cdot \tag{28}$$

Setting $\tau=\tau_1+i\tau_2$, the equation of the saddle-point contour is easily shown to be given by

$$\tau_1 = \tau_0 \mp \cos^{-1}(\operatorname{sech} \tau_2) \quad \text{for} \quad \tau_2 \gtrless 0. \tag{29}$$

The pole of the integrand in (27) is seen to occur at $P:\tau_1=0$, $\tau_2=\cosh^{-1}(\epsilon_1/\sqrt{\epsilon})$. For $x=0$, $\tau_0=\pi/2$ and for this case, the contour C can be deformed into the saddle-point contour without crossing the pole. On the other hand, for $x=\infty$ $\tau_0=0$, and the original contour C crosses the pole P, and the residue of the integral at this pole must be added to the contribution from the saddle point. The saddle contour corresponding to the saddle point

$$\tau_0(P) = \cos^{-1} \frac{\epsilon}{\sqrt{\epsilon_1}} \tag{30}$$

is seen to pass through the pole P. If $\tau_0<\tau_0(P)$, the pole is crossed and the surface wave occurs; if $\tau_0>\tau_0(P)$, there is no surface wave. Thus the region of physical space where the surface wave is present is obtained from (28), (30) and (7):

$$z + d < \frac{\epsilon_2}{\sqrt{\epsilon}} x. \tag{31}$$

The region of the physical space where the surface wave is present is shown in Fig. 1. If the impressed static field is zero, $\epsilon_2=0$. From (31) and Fig. 1, it is seen then that there is no region where the surface wave exists.

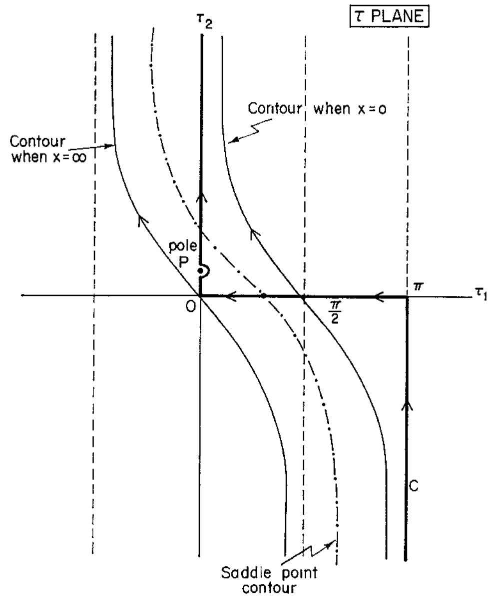


Fig. 3—Integration countours in the τ plane, $x>0$.

Radiation Pattern

In order to obtain the radiation pattern, it is necessary to evaluate the integrals in (26) and (27) asymptotically. For this purpose, let

$$x = \rho \cos \theta; \qquad z = \rho \sin \theta. \tag{32}$$

With (32), (26) and (27) may be rewritten as

$$H_y{}^i(x, z) = \frac{1}{2\pi}\int \frac{\omega\epsilon_0\epsilon}{2\epsilon_1} e^{-ikd \sin \tau} e^{ik\rho \cos (\theta-\tau)} d\tau \text{ for } z > d \tag{33}$$

and

$$H_y{}^r(x, z) = \frac{1}{2\pi}\int \frac{\omega\epsilon_0\epsilon}{2\epsilon_1} \frac{\epsilon_1 \sin \tau + i\epsilon_2 \cos \tau}{\epsilon_1 \sin \tau - i\epsilon_2 \cos \tau} \cdot e^{ikd \sin \tau} e^{ik\rho \cos (\theta-\tau)} d\tau. \tag{34}$$

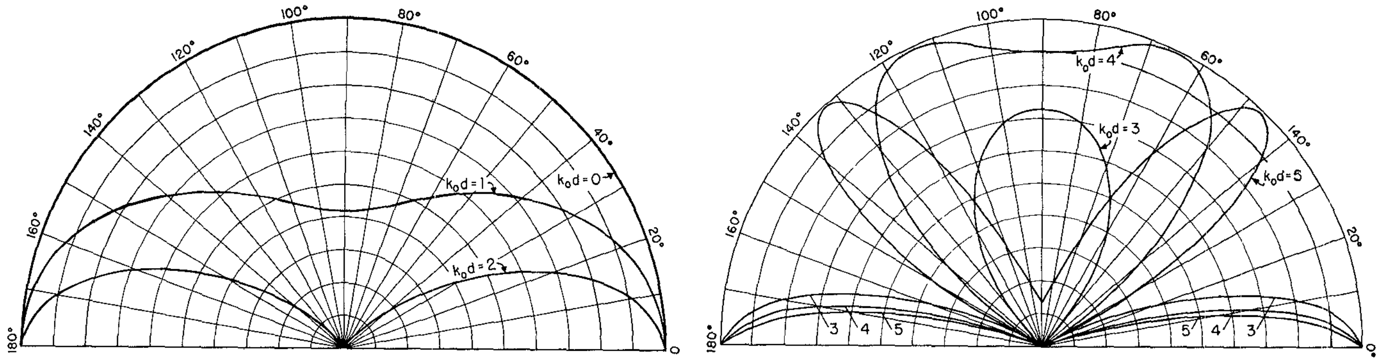
For $k\rho \gg 1$, (33) and (34) are easily evaluated asymptotically with the result that

$$H_y(x, z) = H_y{}^i(x, z) + H_y{}^r(x, z) = \frac{\omega\epsilon_0\epsilon}{2\epsilon_1} \frac{e^{i(k\rho-\pi/4)}}{\sqrt{2\pi k\rho}} \cdot \left[e^{-ikd \sin \theta} + \frac{\epsilon_1 \sin \theta + i\epsilon_2 \cos \theta}{\epsilon_1 \sin \theta - i\epsilon_2 \cos \theta} e^{ikd \sin \theta} \right]. \tag{35}$$

With (6) and (32), it may easily be shown that for $k\rho \gg 1$,

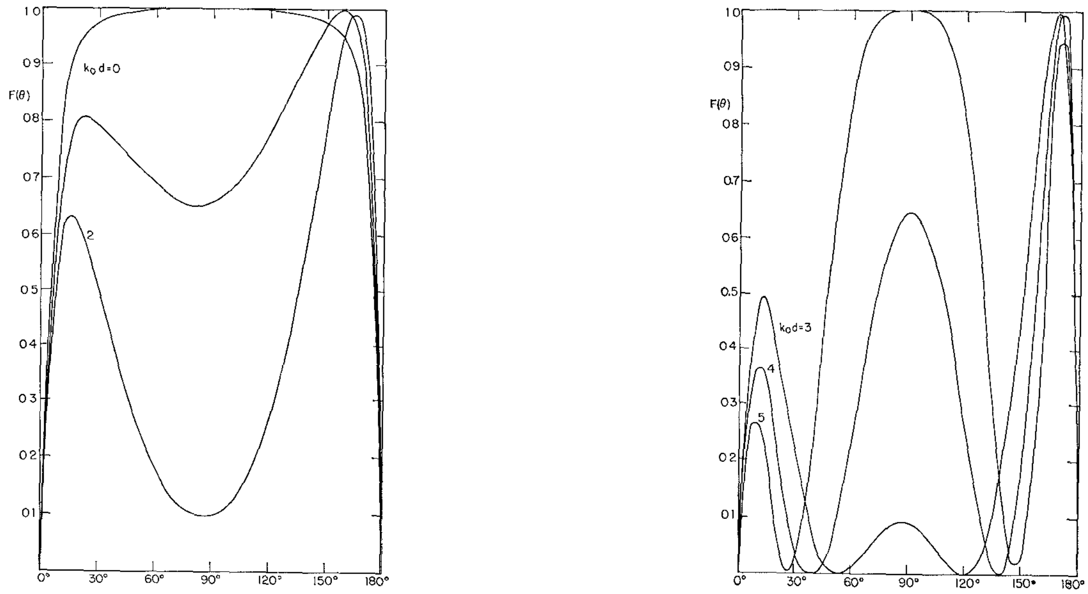
$$E_\theta(\rho, \theta) \simeq -\frac{\epsilon_1 k}{\omega\epsilon_0\epsilon} H_y(\rho, \theta). \tag{36}$$

The outward power flow, per unit area, per unit length of the screen at an angle θ is obtained from (35) and



(a) (b)

Fig. 4—Radiation pattern $\omega_p/\omega = 0.5$, $\omega_c/\omega = 0$.



(a) (b)

Fig. 5—Radiation pattern $\omega_p/\omega = 0.5$, $\omega_c/\omega = 0.25$.

(36) to be

$$S = \frac{1}{2}\,\mathrm{Re}\,\hat{\rho}\cdot \boldsymbol{E}(\rho,\theta)\times \boldsymbol{H}^*(\rho,\theta) = \frac{k\epsilon_1}{2\omega\epsilon_0\epsilon}\,|\,H_y(\rho,\theta)\,|^2$$

$$= \frac{\omega\epsilon_0\epsilon}{8\pi\epsilon_1\rho}\,F(\theta) \tag{37}$$

where

$$F(\theta) = \frac{[\epsilon_1 \sin\theta \cos(kd\sin\theta) - \epsilon_2 \cos\theta \sin(kd\sin\theta)]^2}{[\epsilon \sin^2\theta + \epsilon_2^2]}. \tag{38}$$

$F(\theta)$ given in (38) is defined to be the radiation pattern.

In Figs. 4 and 5, the radiation pattern $F(\theta)$ is plotted for two different sets of values of ω_p/ω and ω_c/ω. In each figure, k_0d is used as a parameter. The patterns are symmetrical about $\theta = \pi/2$, when $\omega_c/\omega = 0$. When $\omega_c/\omega \neq 0$, the plasma is anisotropic and the patterns are no longer symmetrical about $\theta = \pi/2$. A reversal in the direction of the magnetic field changes the sign of ϵ_2 and hence causes the radiation pattern to be rotated through 180° about the z axis.

Efficiency of Excitation

The efficiency of excitation of surface waves is defined to be the ratio of the power propagated as a surface wave per unit width of the screen to the sum of the powers in the surface wave and the radiation fields. The power per unit width of the screen P_R delivered by the magnetic current line source to the radiation field is easily seen to be given by

$$P_R = \int_0^\pi S\rho d\theta = \frac{\omega\epsilon_0\epsilon}{8\pi\epsilon_1}\int_0^\pi F(\theta)d\theta. \tag{39}$$

The power in the surface wave per unit width of the screen is obtained from the relation

$$P_S = \int_0^\infty \hat{x}\cdot\frac{1}{2}\,\mathrm{Re}\,\boldsymbol{E}(x,z)\times\boldsymbol{H}^*(x,z)dz, \tag{40}$$

where $\boldsymbol{H}(x, z)$ and $\boldsymbol{E}(x, z)$ are given, respectively, in (21) and (23). The substitution of $\boldsymbol{E}(x, z)$ and $\boldsymbol{H}(x, z)$

from (23) and (21) in (40) gives

$$P_S = \tfrac{1}{4}\omega\epsilon_0 \left| \epsilon_2 \right| e^{-2kd(|\epsilon_2|/\sqrt{\epsilon})}. \tag{41}$$

The efficiency of excitation obtained from (39) and (41) is

$$\eta = \frac{P_S}{P_S + P_R} = \frac{e^{-2k_0d(|\epsilon_2|/\sqrt{\epsilon_1})}}{e^{-2k_0d(|\epsilon_2|/\sqrt{\epsilon})} + \dfrac{\epsilon}{2\pi\epsilon_1|\epsilon_2|}\displaystyle\int_0^\pi F(\theta)d\theta} \cdot \tag{42}$$

When $k_0d=0$, $\int_0^\pi F(\theta)d\theta$ can easily be evaluated and an explicit expression obtained for η. From (38), for $k_0d=0$, it is seen that

$$F(\theta) = \frac{\epsilon_1^2 \sin^2\theta}{[\epsilon \sin^2\theta + \epsilon_2^2]} \cdot \tag{43}$$

It is readily shown that for $F(\theta)$ given in (43)

$$\int_0^\pi F(\theta)d\theta = \frac{\pi\epsilon_1}{\epsilon}(\epsilon_1 - |\epsilon_2|). \tag{44}$$

The substitution of (44) in (42) yields the following result for $k_0d=0$:

$$\eta = \left[\frac{1}{2} + \frac{\epsilon_1}{2|\epsilon_2|}\right]^{-1}. \tag{45}$$

From (45) and (4), it is obvious that for $k_0d=0$, the efficiency of excitation η increases as ω_c/ω increases.

For other values of k_0d, the value of

$$\int_0^\pi F(\theta)d\theta$$

has to be obtained by numerical integration and the value of η is then determined from (42). For one set of values of ω_p/ω and ω_c/ω, the values of η are given in Table I for different values of k_0d. It is found that the

TABLE I

Values of η for $\omega_p/\omega=0.5$ and $\omega_c/\omega=0.25$

k_0d	0	1	2	3	4	5
η	0.17	0.17	0.25	0.31	0.23	0.13

maximum value of η does not occur when $d=0$, but at some other value of d. Hence, it follows that by a suitable adjustment of the distance of the line source from the ground screen, the power delivered to the radiation field may be minimized and a maximum value obtained for the efficiency of excitation.

Approximate Evaluation of $H_y(x, z)$

The integral representation of $H_y(x, z)$ given in (20a) and (20b) shows that the contribution to $H_y(x,z)$ arises from a pole and a branch-cut integral. The residue at the pole $\zeta=k\epsilon_1/\sqrt{\epsilon}$ gives the surface-wave field and this has been evaluated before. It is now desired to obtain the contribution from the branch-cut integral. This contribution can be obtained as a series in inverse powers of x by expanding the integrand in a Taylor series and integrating term by term. The result of a straightforward calculation gives

$$H_y(x, z) = \frac{k\omega\epsilon_0\epsilon}{\sqrt{2\pi}\,\epsilon_2} e^{i(kx-\pi/4)}\left[\frac{F_1(z, d)}{(kx)^{3/2}} - \frac{3F_2(z, d)}{8(kx)^{5/2}}\right] \tag{46}$$

where

$$F_1(z, d) = z + d - \frac{\epsilon_2}{\epsilon_1}dzk - \frac{\epsilon_1}{\epsilon_2 k} \tag{47}$$

and

$$F_2(z, d) = \frac{7}{k}\left(\frac{\epsilon_1}{\epsilon_2}\right) - \frac{8}{k}\left(\frac{\epsilon_1}{\epsilon_2}\right)^3 + \left[8\left(\frac{\epsilon_1}{\epsilon_2}\right)^2 - 3\right](z+d) - \frac{\epsilon_2}{\epsilon_1}kdz - 4k\frac{\epsilon_1}{\epsilon_2}(d+z)^2 - \frac{4k^2}{3}(d+z)^2 - \frac{4}{3}\frac{\epsilon_2}{\epsilon_1}k^3dz(d^2+z^2). \tag{48}$$

The conditions for validity of the asymptotic series in (46) are

$$x \gg d + z; \qquad kx \gg \frac{\epsilon}{\epsilon_2^2} \cdot \tag{49}$$

Observe that (20a) and (20b) and the condition equation (49) are symmetrical in z and d. It follows that (46) is valid for $z>d$ or $z<d$.

The asymptotic series in (46) represents the radiation field. If the height d of the line source from the ground screen is such that $d=d_m=\epsilon_1/\epsilon_2k$, then it follows that $F_1(z, d_m)=0$ and

$$H_y(x, z) = -\frac{3k\omega\epsilon_0\epsilon}{8\sqrt{2\pi}\,\epsilon_2} F_2(z, d_m)\frac{e^{i(kx-\pi/4)}}{(kx)^{5/2}}, \tag{50}$$

where

$$F_2(z, d_m) = 4\left[1 - \frac{2}{3}\left(\frac{\epsilon_1}{\epsilon_2}\right)^2\right]\left[\frac{1}{k}\left(\frac{\epsilon_1}{\epsilon_2}\right) - z\right]. \tag{51}$$

If the line source is at a distance d_m from the ground screen, the first term in the asymptotic expansion of the radiation field vanishes and the radiation field near the ground screen becomes very weak. An almost pure surface wave may be said to be generated in this special case. This situation is similar to that observed by Cullen in his treatment of the excitation of surface waves on a reactive surface.

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